Chinese/English Journal of Educational Measurement and Evaluation | 教育测量与评估双语季刊

Volume 2 | Issue 3 Article 3

2021

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Recommended Citation
Jiang, Chunlian; Kim, Do-Hong; and Wang, Chuang (2021) "Analysis of a Mathematical Problem-Solving Test on Speed and Students' Strategies: A Mixed Item Response Theory Approach," *Chinese/English Journal of Educational Measurement and Evaluation | 教育测量与评估双语季刊*: Vol. 2 : Iss. 3 , Article 3.
Available at: https://www.ce-jeme.org/journal/vol2/iss3/3

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Analysis of a Mathematical Problem-Solving Test on Speed and Students' Strategies: A Mixed Item Response Theory Approach

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Abstract
The present study used the mixed item response theory (IRT) model to identify qualitatively distinct subgroups of sixth-grade students with respect to their performance on word problems on speed. A total of 345 Singaporean students and 361 Chinese students took a problem-solving test on speed. The mixed IRT analysis revealed two latent classes — the algebra proficient group and the algebra novice group. The algebra proficient group was more likely to use traditional algebraic and arithmetic strategies to solve the problems, whereas the algebra novice group was more likely to use model drawing, unitary, and guess-and-check strategies, in addition to using traditional arithmetic and algebraic strategies. Findings of the study indicate that a greater variety of problem-solving strategies could be encouraged in upper primary schools to help students make connections among these strategies, in particular, between these strategies and the abstract algebraic strategies, and finally to achieve a successful transition from arithmetic to algebra learning.

Keywords
Word problem;
Problem-solving strategies;
Mixed IRT model;
Arithmetic strategies;
Algebraic strategies

Introduction

Since the 1980s, mathematical problem solving has been one of the main topics in mathematics education (NCTM, 1989). It has become one of the objectives of mathematics curricula in almost all countries around the world. For example, it was incorporated into Singapore syllabi for both primary and secondary schools in 1992 (Ministry of Education, 1990a, 1990b) and is still the central focus of the pentagonal framework of the Singapore mathematics curriculum (Curriculum Planning and Development Division [CPDD], 2012, 2019, 2020). Developing students' abilities to solve application problems has always been one of the objectives of China's mathematics curricula at both primary and secondary levels (National Institute for Curriculum and Textbook Research, 1999). Word problems, as a bridge linking the real world and the mathematical world, are widely used to promote students' understanding of mathematical concepts and to develop their abilities to solve mathematical problems (Verschaffel & De Corte, 1997). Speed word problems are such a kind of word problems that most frequently appeared in mathematics textbooks (Mayer, 1981). Our previous study investigated the measurement properties of a mathematical problem-solving test on speed (Jiang et al., 2018), and the results revealed the misfit of some items to the unidimensional Rasch model and differential item functioning (DIF) between Singaporean and Chinese students. The present study used a mixed item response theory (IRT) model to determine whether two or more distinct latent classes could be identified, which differed with respect to parameter estimates and strategy use.

1 Literature Review

1.1 Mathematical Problem Solving in Mathematics Curricula in Singapore and China

Mathematical problem solving is at the center of the Singapore mathematics curriculum framework (CPDD, 2019, 2020). One of the three aims of mathematics education in Singapore is to enable students to develop cognitive and metacognitive skills through a mathematical approach to problem solving (CPDD, 2012). Heuristics,

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which are general rules of what students can do to tackle a problem (e.g., "using a representation" or "making a guess"), are suggested in the mathematics syllabus for primary schools (CPDD, 2012). A unit entitled "Solving word problems" is often included at the end of a chapter in primary mathematics textbooks (Lee et al., 2018a, 2018b).

Mathematical problem solving also plays an important role in the mathematics curriculum in China (Cai & Nie, 2007). Research on mathematical problem solving often focuses on the study of multiple solutions of one problem, the use of multiple problems to teach one concept, and the discussion of multiple variations of one problem to help students form a coherent scheme of an important concept including speed (Cai & Nie, 2007; Jiang et al., 2014). In an analysis of 264 articles published in educational journals in Macao in 2009-2018 and were related to mathematics education, we found that nearly half (48.5%) were related to mathematical problem solving; such a percentage was much higher than that related to mathematics curriculum (15.5%) and instruction (22.7%) (Cheung et al., under review).

It seems that the Singapore mathematics curriculum developers adopted the "teaching of mathematical problem solving" approach, whereas China's mathematics curriculum developers used a "teaching mathematics through mathematical problem solving" approach. The two different approaches might provide different learning opportunities and experiences for their students. Our previous comparative study (Jiang & Chua, 2010; Jiang et al., 2014) found that the Singaporean students used more categories of strategies (e.g., model drawing method, guess-and-check, and unitary methods) for solving word problems. They might benefit from the teaching of a greater variety of problem-solving heuristics included in their mathematics curriculum (Fan & Zhu, 2007). In comparison, Chinese students were more likely to use the traditional arithmetic and algebraic strategies, and their selection of strategy categories was a bit limited. This is probably because they learned algebra earlier, and there were no such clear requirements regarding the teaching of different kinds of problem-solving strategies in China as in Singapore.

1.2 Word Problems on Speed in Mathematics Curricula in Singapore and China

Word problems are often used to develop students' problem-solving skills. Word problems on speed were frequently included in mathematics textbooks. Mayer (1981) found that motion (i.e., speed) problems took up 12% (133/1097) of algebra word problems that were selected from ten standard algebra textbooks used in U.S. lower secondary levels. Moreover, the number of motion problems was the highest among the 21 problem families including motion, current, work, unit cost, coins, and so on. Word problems on speed are also frequently included in international studies such as the Trends in International Mathematics and Science Study (TIMSS) (Mullis et al., 2012) and Programme for International Student Assessment (PISA) (Organization for Economic Co-operation and Development [OECD], 2014). Word problems on speed were selected for the present study because they are application problems of various mathematical concepts from primary to university levels (Jiang, 2005, 2016).

Word problems on speed have been presented in the textbooks used in many countries. In Singapore, up to 3-step word problems involving speed and average speed are included in sixth-grade mathematics as a unit (CPDD, 2012). Word problems about speed are presented under the unit of ratio in seventh-grade mathematics (CPDD, 2019).

In China, word problems about speed have been presented in different units from time to time to illustrate the applications of various mathematical topics in the real world. For example, they are first presented in the second-grade mathematics textbooks when multiplication and division are taught to eight- to nine-year-old students (People's Education Press, 2013). Since then, various word problems about speed are presented in third- to sixth-grade mathematics textbooks when four operations on whole numbers, decimals, and fractions are taught. Word problems about speed that can be solved using linear equations are presented in sixth and seventh grades when solving linear equations is covered. Word problems about speed that can be solved using fractional expressions are presented in eighth grade when fractional expressions are taught.

Word problems about speed are treated differently in the Chinese and Singapore mathematics curricula, which may bring about differences in student performance between the two countries. Our previous comparative study between sixth-grade Singaporean and Chinese students (Jiang et al., 2014) found that the Singaporean students performed better in one problem involving fractions, whereas the Chinese students performed better in seven problems with medium-to-large effect sizes in the test. Most of the problems in which the Chinese students performed better were algebraic problems. In the problem that Singaporean students performed better, the students successfully used model drawing and unitary methods to transfer the multiplication of two fractions into the multiplication of whole numbers, which greatly reduced the difficulty level of the problem (Jiang & Chua, 2010; Jiang et al., 2014).

1.3 Studies Involving Word Problems on Speed

A number of studies have included problems on speed as a specific rate model for multiplication and division (e.g., Greer, 1992; Lamon, 2007). However, the problems used in previous studies belong only to the simplest of the 13 motion problem categories identified by Mayer (1981). Our previous study investigated the psychometric properties of a 14-item math test that included more diverse and more complex word problems on speed based on the Rasch's IRT model (Jiang et al., 2018). Our analysis revealed the misfit of some items to the unidimensional Rasch model and DIF between Singaporean and Chinese students. Building on these findings, the present study was designed to determine whether there existed two or more distinct sub-populations (latent classes) which would differ with respect to parameter estimates, resulting in measurement non-invariance. We also examined whether there were differences in the use of strategies between the students in the resulting latent classes, which, in turn, could identify potential sources of DIF. The differences might shed light on the impact of the instruction of mathematical problem solving on student performance.

If measurement invariance is violated, differences in scores between the Singaporean and Chinese students may not reflect true differences in the mathematical concepts being measured. Predominant measurement models such as factor analysis and item response theory (IRT) assume that a single measurement model can be applied to all individuals in a population. Measurement invariance studies using these approaches often use manifest groups such as gender or ethnicity. Limitations of these approaches are that group membership must be known and it may not adequately model heterogeneity due to unknown characteristics. Mixed IRT models can be used to investigate heterogeneous subpopulations that are qualitatively distinct with respect to a measurement model. For example, mixed Rasch models (Rost, 1990) extend the more traditional IRT models to take heterogeneity in the population into account, allowing for different performance levels within latent classes. Furthermore, in a mixed Rasch model, a unidimensional Rasch model holds within each latent class but with different sets of item difficulty parameters across the latent classes (Rost, 1990). Such kind of analysis might help us understand why students respond differentially to test items and better understand potential sources of DIF.

This study was guided by the following two research questions:

1. How many latent classes can be found based on the students' performance on a mathematical problem-solving test on speed?

2. How do students in the identified latent classes differ in terms of their uses of problem-solving strategies?

2 Method

2.1 Test Instrument

The instrument is a test that includes 14 word problems on speed (Appendix). It was developed based on Mayer's (1981) classification of word problems on speed and an analysis of word problems on speed in the textbooks used in China and Singapore. The following paragraphs provide brief descriptions of the 14 problems. The terms "problems" and "items" are used interchangeably in this study because each item in the test is a word problem.

Problems 1-3. These three problems describe only one motion of an object. For a single motion, three variables — distance (D), speed (S), and time (T) — are involved and their relationship can be described as $D = S \times T$. Given any two of them, the third can be set as the unknown. Therefore, these are the basic word problems about speed.

Problems 4 and 9. These two problems describe two motions of an object where the directions of the two motions can be assumed to be the same. Problem 4 can be solved using arithmetic strategies. Problem 9 is a typical algebraic word problem like the Chickens and Rabbits Problem in the ancient Chinese mathematics book *Sunzi Suanjing* (Horng, 2012).

Problems 8, 13, and 14. These three problems describe a round trip, where one object makes two motions with the same distance but in opposite directions. Problem 8 can be solved using arithmetic strategies, whereas Problems 13 and 14 cannot be solved using the same methods. The knowledge of inverse proportions could be used to obtain a solution to Problem 13 but not to Problem 14 (Jiang, 2009).

Problems 6, 10, and 11. These three problems describe two motions of two objects. In Problems 6 and 10, the two objects are moving towards each other from two different points; in Problem 11, they are moving in the same direction with one ahead of the other. Arithmetic strategies can be used to solve Problem 6 but not Problems 10 and 11 if a student does not know the formulae. Problem 10 asks for the time taken for the two objects to meet; Problem 11 asks for the time taken for one to catch up with the other.

Problems 5, 7, and 12. These three problems describe three motions of one object, where the directions of the three motions can be the same. They also involve fractions to represent the relationships between distances of individual parts of the journey to the entire journey or to the remaining journey after the first motion. This kind of problem was found in a popular workbook written by Fong (1998). They were included to examine whether the students could apply the concept of average speed of two motions to three motions of an object. The results from our previous study indicated that students could apply such a concept (Jiang & Chua, 2010; Jiang et al., 2014).

Problems 1-3 are short answer questions. Problems 4-14 are open response questions that ask participants to write down their working process in the space provided below. As previously stated, Problems 4-8 are arithmetic problems, and Problems 9-14 are algebraic problems.

2.2 Participants

A sample of 706 sixth graders from China and Singapore participated in the study. The 361 Chinese students were from three primary schools in Wuhan and the 345 Singaporean students were from four primary schools in Singapore. Wuhan is the capital city of Hubei Province, located in Central China. Schools were selected from all three districts in Wuhan. The schools were recommended by an officer who had worked with the Hubei Provincial Department of Education for five years. In Singapore, classes were selected from each school to reflect the average academic level of the sixth graders in the school after consultation with the principals about the mathematics performance of each class. No calculators were allowed. It took an average of 80 minutes for the students to finish the test. All students had already learned the topic of speed at the sixth-grade level before the administration of the test.

2.3 Scoring System

Items 1-3 were short-answered questions. Students' responses to these items were scored dichotomously with 1 for a correct answer and 0 for an incorrect answer. The other 11 items (Items 4-14) were rated on a 5-point scale from 0 to 4. A student would receive a grade of 4 if the student's answer was correct and with an appropriate solution process. A student would receive a grade of 3 if the answer was correct but incomplete with 75% of the solution process. If a student showed 100% of the solution process, but the solution process contained errors in computation, the solution was also graded as 3. If a student's solution included 50% of the correct process, it was graded as 2. If a student's solution included some (less than 50%) correct steps, it was graded as 1. If a student's answer showed no understanding of the problem, it was graded as 0. A blank response also received 0 points.

2.4 Data Analysis

The mixed Rasch model was used in this study. The fit of the mixed Rasch model of different class solutions was evaluated by Akaike's Information Criterion (AIC), Schwartz's Best Information Criterion (BIC), and Bozdogan's Consistent AIC (CAIC) implemented in the program WINMIRA (von Davier, 2001). The model with the smallest fit index value was selected. Beyond relying on statistical criteria, we also considered the relative sizes of the latent classes and the magnitude of the parameter differences across latent classes. Any improvement in model fit was evaluated in relation to its substantive significance.

Once the best fitting model was identified, item difficulties and person estimates within each class were examined to characterize the latent classes. The item Q-index (Rost & von Davier, 1994) was used to examine the fit of the items within each latent class. The Q-index ranges from 0 to 1, with 0 indicating perfect fit (perfect item discrimination), 1 indicating perfect misfit (perfect negative item discrimination), and .5 indicating random response behavior. The standardized Q-index (a z-statistic) was used to determine whether the item pattern deviation from the model was statistically significant. The class membership probabilities for each student were also estimated. Based on these probabilities, each student was assigned to the latent class for which his or her membership probability was the highest.

We also examined the strategies used by the latent classes for solving problems 4 to 14 (the first three problems were short-answer problems). Five specific categories of problem-solving strategies were identified (Jiang et al., 2014): arithmetic, algebraic, model-drawing, guess-and-check, and unitary strategies. A no-strategy category was included for responses where the student wrote nothing or only copied information from the problem statement without any further work. The descriptions of the six strategies are included in Table 1. The percentage of students (SP) in each class using each strategy and the

Table 1
Descriptions of Problem-Solving Strategies

Strategy category	Description
Arithmetic strategy	The student writes down a mathematical statement involving one or more arithmetic operations on the numbers given in the problem.
Algebraic strategy	The student chooses one or more unknowns as variables and sets up one or more equations.
Model drawing strategy	The solution is suggested by or follows a model or a diagram.
Guess-and-check	The student uses the following process: (a) Make a guess of an answer or the unknown in the problem based on an estimation; (b) Check if the constraints given in the question or implied from some of the question statements are satisfied. If all the constraints are satisfied, the guess is correct; the answer has been obtained or can be worked out. All processes will end at this point. If the constraints are not satisfied, the guess will be refined or adjusted, and another guess will be made followed by another round of guess-and-check.
Unitary strategy	The student finds the value equivalent to one unit of a quantity from an equivalence statement and obtains the value equivalent to more units of the quantity using the value for one unit just found.
No strategy	Absence of a written response or only pieces of information taken from the question are written down without any further work.

success rate (SR), defined as the percentage of students who used a specific strategy to reach the correct answer, were obtained in order to explain the difficulty levels of the items.

Among the six strategies, only the unitary strategy might be unfamiliar to us. Therefore, three solutions using this strategy for solving item 7 are given in Figure 1. In solving Item 7, we can draw the intermediate conclusion that "$\frac{4}{7}$ of the total distance is 36 kilometers" based on the givens of the problem, and then we use $36 \div 4/7$ to find the total distance of the whole journey. When using the unitary strategy suggested in mathematics textbooks in Singapore, a student could first write a mathematical sentence like "4 units = 36 km." The student then could find the distance for one unit and the distances of all the individual parts of the journey. The use of unitary strategy could help students avoid difficulties with divisions involving fractions. A similar process can also be used to help students understand why we need to turn the divisor upside down when divided by a fraction. This is also the method that is used to deal with the division problem with a fraction divisor in the mathematics textbooks in Singapore. Similar approaches are also used for problems with ratios and proportions to remove students' learning difficulties.

The sixth-grade students are at the transition stage from arithmetic thinking to algebraic thinking. Model drawing, unitary, and guess-and-check strategies are often used to help students to achieve a successful transition (Fong, 1994; Jiang, 2005). The strategy analysis will help us to better understand students at the transition stage and provide insights into the improvement of instructional design.

3 Results

3.1 Model Fit and Item Difficulty Estimates

The fit values are listed in Table 2. Although the one-class model had the smallest CAIC value, the two-class model had the smallest AIC and BIC indices. On the basis of evaluating the statistical fit indices, the two-class model with class sizes .57 and .43 appeared to fit the data the best. The mean person measure WLE (Warm's modified likelihood estimates) was higher for Class 1 (M = 1.81, SD = 1.01) than for Class 2 (M = 0.98, SD = 0.57). There were 422 students in Class 1, among which 335 were from China and 87 from Singapore. There were 284 students in Class 2, among which 258 were from Singapore and 26 from China.

Table 3 shows the means and standard deviations of item raw scores by the two classes. Class 1 performed significantly better than Class 2 on ten items. Class 2 performed better than Class 1 on two items; however, the differences did not reach a significant level. As previously

Figure 1
Three Solutions Using Unitary Strategies for Item 7

Solution 1	Solution 2	Solution 3
$72 \times \frac{1}{2}$ = 36km 4 units = 36 km 1 unit = 9 km Total distance TD = 7 units = 63 km	$72 \times \frac{1}{2}$ = 36 km $\frac{12}{21}$ of the journey = 36 km $\frac{1}{21}$ of the journey = $36 \div 12$ = 3 km $\frac{21}{21}$ of the journey = 3×21 = 63 km	$72 \times \frac{1}{2}$ = 36 km 2 units = 36 km 1 unit = 18 km 3 units = 54 km 6 units = 54 km 1 unit = 9 km Total distance TD = 7 units = 63 km

Total time TT = $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1\frac{1}{2}$ hours. Average speed AS = $63 \div 1\frac{1}{2}$ = 42 km/h.

Table 2
Model Fit Indices for One-, Two-, and Three-Class Models

Model	AIC	BIC	CAIC
One class	14625.82	14885.72	14942.72
Two classes	14326.04	14850.40	14965.40
Three classes	14351.77	15140.58	15313.58

Note. AIC = Akaike Information Criterion; BIC = Bayesian Information Criterion; CAIC = Consistent Akaike Information Criterion.

mentioned, items 1-8 were all arithmetic items and items 9-14 were algebraic items. Class 1 students performed significantly better on all the algebraic items than Class 2 students, but their performances were not significantly different on half of the arithmetic items. Therefore, we named Class 1 as the Algebra Proficient (AP) Class and Class 2 as the Algebra Novice (AN) Class.

Table 4 shows the class-specific Rasch item difficulties and item fit indices (Q-index). The standardized Q-indices indicated that all the items fit well for both classes. The Q values for all polytomously scored items (items 4-14) were close to zero, ranging from .04 to .24 for the AP Class and from .16 to .22 for the AN Class. The Q values for three dichotomously scored items (items 1-3) were relatively large as shown in Table 2, but the standardized Q-indices were not statistically significant, indicating adequate fit.

Table 3
Class-Specific Mean Score and Standard Deviation for Each Item

Item	Class 1 ($n = 422$)		Class 2 ($n = 284$)		t
	M	SD	M	SD	
Item 1	0.99	0.084	0.94	0.231	4.00***
Item 2	1.00	0.069	0.99	0.102	0.90
Item 3	0.99	0.084	0.97	0.175	2.49*
Item 4	3.59	1.063	3.70	0.782	−1.52
Item 5	3.53	1.067	3.48	0.967	0.73
Item 6	3.47	1.098	2.64	1.349	9.01***
Item 7	3.05	1.378	3.19	1.132	−1.41
Item 8	2.83	1.597	2.19	1.476	5.35***
Item 9	3.12	1.584	2.09	1.867	7.88***
Item 10	3.90	0.383	1.11	1.600	34.45***
Item 11	3.35	1.346	1.50	1.472	17.31***
Item 12	2.08	1.696	1.29	1.180	6.79***
Item 13	2.20	1.947	0.88	1.616	9.41***
Item 14	1.68	1.945	0.27	0.973	11.25***

*$p < .05$; ***$p < .001$.

Table 4
Item Difficulty Estimates and Q-Indices Arranged in Order of Decreasing Difficulty

Class 1			Class 2		
Item	Item difficulty	Q-index	Item	Item difficulty	Q-index
Item 14	2.22	.04	Item 14	1.87	.22
Item 12	1.89	.07	Item 13	1.46	.22
Item 13	1.87	.03	Item 10	1.37	.16
Item 8	1.26	.09	Item 12	1.29	.22
Item 9	1.18	.07	Item 11	1.05	.22
Item 7	1.08	.06	Item 9	0.90	.18
Item 11	0.91	.08	Item 8	0.69	.16
Item 6	0.61	.12	Item 6	0.46	.21
Item 4	0.54	.12	Item 7	0.05	.22
Item 5	0.52	.09	Item 4	−0.37	.22
Item 10	−1.59	.24	Item 5	−0.38	.22
Item 1	−3.21	.23	Item 1	−2.05	.55
Item 3	−3.46	.79	Item 3	−2.60	.57
Item 2	−3.82	.75	Item 2	−3.74	.69

The item difficulty measures of the 14 items (except items 8, 10, and 11) for the two classes were more or less within expectations. Items 1-3 were the easiest, four arithmetic items (items 4-7) were at the intermediate difficulty level, and four algebraic items (items 9, 12-14) were at the high difficulty level. In particular, item 14 was found to be the most difficult item for both classes.

There was a noticeable discrepancy in the item difficulty order between the two latent classes for items 8, 10, and 11. The difficulty estimate of item 8 for the AP Class was just below items 12-14. However, for the AN Class, it was at the intermediate difficulty level, close to items 4-7. The item difficulty estimate of item 10 for the AP Class was quite low and negative, indicating a relatively easy item. Item 10, however, was the third most difficult item for the AN Class. Item 10 also showed the largest difference in item difficulty (2.96 logits) between the two classes. Our previous study (Jiang et al., 2018) using the traditional Rasch model indicated that the item fit for item 10 was poor. Item 10 was also flagged as having DIF between the Singaporean and the Chinese students. For the AP Class, item 11 was estimated to be easier than items 7-9, whereas for the AN Class, item 11 was estimated to be more difficult than items 7-9. The following section on the strategy analysis for items 4-14 might provide some explanations for the discrepancy in item difficulty between the latent classes.

3.2 Differences in the Use of Problem-Solving Strategies Between the Latent Classes of Students

In this section, we examined the mathematical problem-solving strategies the two classes used. We first summarized the differences in strategy use between the two classes for the 11 open-response items (items 4-14). We then examined the strategies used for individual items in more details to illustrate the differences between the two classes.

The percentages of cases (number of items [11] × number of students) where each class of students used each strategy are presented in Table 5. These results provide an overarching picture of the significant differences in strategy use between the two classes. Both classes used arithmetic strategies the most frequently; the students in the AP Class used them in a higher percentage of cases. For the AP Class, algebraic strategies were used the second most frequently. For the AN Class, model drawing was the second most frequently used strategy, followed by unitary and guess-and-check methods. The z-scores in the last column indicated that the two classes were significantly different in the percentages of students using the five strategies. However, there was no significant difference in the percentages of students having no strategies between the two classes.

Table 5
Class-Specific Overall Strategy Use Across Items 4-14

Strategy category	AP class (n = 4,642)	AN class (n = 3,124)	z
Arithmetic strategy	65.92[a]	52.08	12.23***
Algebraic strategy	12.88	0.99	18.83***
Model drawing strategy	4.63	17.99	−19.24***
Guess-and-check	5.60	10.69	−8.28***
Unitary strategy	2.46	10.79	−15.39***
No strategy	8.51	7.46	1.66

[a]The numbers were the percentages (% is omitted) of the cases (number of items [11] × number of students) where the class of students used the specific kind of strategies.
***$p < .001$.

Based on the item difficulty measures in Table 4, we classified items 4-14 into the following three groups: (a) items 4-7; (b) items 9 and 12-14; and (c) items 8, 10, 11. For each group, we examined differences in the use of strategies between students of the two latent classes.

Items 4-7. The percentages of students in each class using the different strategies for solving items 4-7 and the success rates are shown in Table 6. Because the results for items 4 and 6 are very similar, the following discussion will group them together, and similarly for the results for items 5 and 7.

For items 4 and 6, the majority of the students in both classes used arithmetic strategies, and the AP Class could use them more successfully than the AN Class. Higher percentages of the students in the AN Class used the model drawing strategy than those in the AP Class. The percentages of students using algebraic strategies and having no strategies were all very low.

For items 5 and 7, the strategy use between the two classes was significantly different (item 5: $\chi^2 = 309.22$; item 7: $\chi^2 = 305.06$; $p < .001$). The students in the AP Class mainly used arithmetic and algebraic strategies. However, for students in the AN Class, the main strategies were model drawing and unitary strategies. A key difference between these two kinds of problem-solving approaches lies in the operations on fractions. In the use of the arithmetic and algebraic strategies, addition/subtraction and multiplication of fractions were involved. However, when the model drawing and unitary strategies were used, these operations were converted into operations with whole numbers instead (Jiang & Chua, 2010; Jiang et al., 2014). These strategies provided the AN Class with an advantage in item 7 even though the performance differences between the two classes were not significant.

Items 9 and 12-14. The percentages of students in each class using different strategies for solving items 9 and 12-14 and the success rates are shown in Table 7.

For item 9, the strategy use between the two classes was significantly different ($\chi^2 = 141.24$, $p < .001$). The success rates revealed that algebraic and guess-and-check strategies were the two effective strategies for this item. The data in Table 7 showed that a higher percentage of students in the AN Class used inappropriate arithmetic strategies ($z = -3.05$, $p < .01$). In the use of two effective strategies, a higher percentage of the students in the AP Class used the algebraic strategies ($z = 11.11$, $p < .01$) than those in the AN Class, whereas a higher percentage of the students in the AN Class used guess-and-check strategies than those in the AP Class ($z = -6.92$, $p < .001$). In total, nearly 70% of the students in the AP Class used the two effective strategies, whereas only about 60% of the students in the AN Class chose to use them. Nearly 4% of the students in the AN Class used model drawing strategies while only two students in the AP Class used it ($z = -3.27$, $p < .01$). The percentages of students having no strategies were similar in both classes.

For item 12, the strategy use between the two classes was significantly different ($\chi^2 = 163.95$, $p < .001$). The students in the AP Class mainly used arithmetic and algebraic strategies. However, for the students in the AN Class, the main strategies were model drawing and unitary strategies. Although the use of model drawing and unitary strategies could convert the division of fractions into whole numbers (Jiang & Chua, 2010), it seemed difficult for the AN Class, which led to a relatively lower success rate compared with the use of all strategies by the AP Class.

In solving items 13 and 14, for the AP Class, the two most frequently used strategies were arithmetic strategies (48.3% and 37%) and algebraic strategies (27% and 29%). Close to 10% of the students in the AP Class used guess-and-check strategies for both items. For the AN Class, the majority of students used arithmetic strategies (66.9% and 57.7%). The students in the AN Class also used guess-and-check (15.5% and 8.8%) and model drawing strategies (5.6% and 8.4%) when solving items 13 and 14. Based on the success rates, algebraic and guess-and-check strategies were two effective strategies for items 13 and 14.

In summary, students in the AP Class performed better than those in the AN Class on the four items because they could use the effective guess-and-check and algebraic strategies, particularly on items 9 and 14. Although higher percentages of students in the AN Class used the model drawing and unitary strategies, their success rates were all very low.

Items 8, 10, and 11. Table 8 shows the percentages of students in each class using the different strategies for solving items 8, 10, and 11 and their success rates.

For item 8, the majority of students in both classes used the arithmetic strategies (83.2% for the AP Class; 82.7% for the AN Class). The second most frequently used strategy for the AP Class was algebraic strategies, whereas it was the model drawing strategy for the AN Class. The students in the AP Class used the arithmetic and algebraic strategies more successfully than those in the AN Class.

Most students in both classes used arithmetic strategies for solving item 10 (AP Class: 94.3%; AN Class: 63.4%)

Table 6

Percentages of Students (SP) in the Two Classes Using Different Strategies and Success Rates (SR) for Items 4-7

		AR		MD		US		AL		NS
		SP	SR	SP	SR	SP	SR	SP	SR	SP
Item 4	AP class	93.8	86.4	2.8	100.0	0.0	—[a]	1.9	87.5	1.4
	AN class	92.3	83.6	7.4	71.4	0.0	—	0.0	—	0.4
Item 6	AP class	86.5	76.4	10.0	88.1	0.0	—	0.2	100.0	3.3
	AN class	70.4	34.0	28.5	60.5	0.0	—	0.0	—	1.1
Item 5	AP class	69.7	82.0	7.6	78.1	12.1	94.1	7.8	72.7	2.8
	AN class	10.9	48.4	35.6	71.3	51.4	76.0	1.8	60.0	0.4
Item 7	AP class	65.6	64.3	10.0	83.3	10.0	83.3	6.4	29.6	8.1
	AN class	10.6	23.3	49.6	60.3	37.0	61.9	1.8	60.0	1.1

Note. AR = arithmetic strategy; MD = model drawing strategy; US = unitary strategy; AL = algebraic strategy; NS = no strategy. SP = percentage of students using a kind of strategy (strategy percentage); SR = percentage of students who used the given strategy and solved the problem correctly (success rate). % was omitted.
[a]No SR could be calculated.

Table 7

Percentages of Students (SP) in the Two Classes Using Different Strategies and Success Rates (SR) for Item 9 and Items 12-14

		AR		MD		US		GC		AL		NS
		SP	SR	SP	SR	SP	SR	SP	SR	SP	SR	SP
Item 9	AP class	18.7	53.2	0.5	50.0	0.0	—[a]	31.5	97.7	38.4	92.6	10.9
	AN class	28.5	6.2	3.9	27.3	0.0	—	57.7	95.7	2.1	83.3	7.7
Item 12	AP class	43.4	39.9	13.7	67.2	4.3	44.4	0.0	—	15.9	61.2	22.7
	AN class	28.2	3.8	33.8	22.9	27.8	7.6	0.0	—	1.8	0.0	8.5
Item 13	AP class	48.3	36.3	2.6	72.7	0.7	66.7	9.2	87.2	27.0	87.7	12.1
	AN class	66.9	4.21	5.6	18.8	2.5	28.6	15.5	90.9	0.4	100.0	9.2
Item 14	AP class	37.0	16.0	1.2	0.0	0.0	—	9.2	92.3	29.1	87.8	23.5
	AN class	57.7	0.0	8.1	0.0	0.0	—	8.8	68.0	1.1	0.0	24.3

Note. AR = arithmetic strategy; MD = model drawing strategy; US = unitary strategy; GC = guess-and-check; AL = algebraic strategy; NS = no strategy. SP = percentage of students using a kind of strategy (strategy percentage); SR = percentage of students who used the given strategy and solved the problem correctly (success rate). % was omitted.
[a]No SR could be calculated.

and item 11 (AP Class: 84.6%; AN Class: 61.3%). However, the AP Class had much higher success rates than the AN Class. The solving procedures provided by the students of the two classes might be quite different. The students in the AP Class used formulae for solving the two items, whereas students in the AN Class made irrelevant operations on the givens (Jiang et al., 2014). For both classes, the second most frequently used strategy was guess-and-check strategies (AP Class: 4.3% for item 10 and 7.3% for item 11; AN Class: 14.4% for item 10 and 21.1% for item 11). The success rates of the AP Class were much higher than those of the AN Class. For the AN Class, the third most frequently used strategy was model drawing strategies (item 10: 7.0%; item 11: 4.9%). Higher

Table 8
Percentages of Students (SP) in the Two Classes Using Different Strategies and Success Rates (SR) for Items 8, 10, and 11

		AR		MD		GC		AL		NS
		SP	SR	SP	SR	SP	SR	SP	SR	SP
Item 8	AP class	83.2	64.4	1.4	50.0	0.0	—[a]	10.9	67.4	4.5
	AN class	82.7	29.8	13.4	55.3	0.0	—	2.1	16.7	1.8
Item 10	AP class	94.3	91.2	0.2	100.0	4.3	100.0	0.9	100.0	0.2
	AN class	63.4	21.7	7.0	0.0	14.4	19.5	0.0	—	15.1
Item 11	AP class	84.6	81.5	0.9	75.0	7.3	83.9	3.1	100.0	4.0
	AN class	61.3	17.2	4.9	14.3	21.1	55.0	0.0	—	12.7

Note. AR = arithmetic strategy; MD = model drawing strategy; GC = guess-and-check; AL = algebraic strategy; NS = no strategy. SP = percentage of students using a kind of strategy (strategy percentage); SR = percentage of students who used the given strategy and solved the problem correctly (success rate). % was omitted.
[a]No SR could be calculated.

percentages of students in the AN Class had no strategies than those in the AP Class. All these factors contributed to the items having relatively higher difficulty levels for the AN Class than for the AP Class.

Item 10 was relatively more difficult than item 11 for the AN Class. This is probably because the computation involved and the answer (2 hours 5 minutes) in item 10 is more complicated than that of item 11 (7 hours).

In summary, the analysis of strategy use by the two classes explained the differences in item difficulty estimates between the two classes. In solving the four arithmetic items (items 4-7), the majority of the AP Class used arithmetic strategies, whereas students in the AN Class used the model drawing and unitary strategies in addition to the arithmetic strategies. In solving the four algebraic items (items 9 and 12-14), higher percentages of students in the AP Class used the guess-and-check and algebraic strategies with high success rates, which brought them better performance. In solving the three items (items 8, 10, and 11) with a discrepancy in item difficulty order between the two classes, the success rates of strategy use by students in the AN Class were all very low. On the other hand, students in the AP Class could use arithmetic strategies with high success rates, leading to the discrepancy in performance and item difficulty.

The z-tests were also conducted to compare the percentages of students having no strategies in the two classes; it was found that there were only significant differences for items 10 ($z = -8.05$, $p < .001$) and 11 ($z = -4.30$, $p < .001$). This is surprising because our previous study indicated that there were significant differences between the percentages of Singaporean and Chinese students having no strategies for almost all items except items 4 and 14 (Jiang et al., 2014). Although there was an overlap between the latent groups and manifest groups, the result obtained in the current study indicated that students' cognitive processes that are involved in mathematical problem solving and lead to group differences in item measures (i.e., measurement invariance, DIF) might not be perfectly explained by the manifest groups.

4 Conclusion, Discussion, and Implications

The results of this study indicated that two latent classes were present in the data. The heterogeneity in measurement revealed in this study provides further explanations for the item misfit and DIF between Singaporean and Chinese students found in the previous study (Jiang et al., 2018). For the two classes, one consisted of algebra proficient students, and the other consisted of algebra novice students. A separate set of item difficulty parameters was required for each class, and their item difficulty orders showed unique patterns beyond what could be obtained using traditional IRT methods. For both classes, items that could be solved by directly using formulae were the easiest. Furthermore, arithmetic items were at the intermediate difficulty level, and algebraic items were at the high difficulty level.

Another contribution of this study is the instrument that could be used in fifth- to seventh-grade mathematics.

Although the study was conducted with sixth-grade students, the easy and the intermediate difficulty items could be used with fifth-grade students and the high difficulty items could be used with seventh-grade students.

This study revealed a big jump in difficulty levels from arithmetic to algebraic word problems on speed, which is significant for mathematics teachers because they need to give students more time in classroom instruction to allow for the transition from arithmetic to algebraic thinking. This is a contribution of the current study to the field of mathematical problem solving.

Mathematics educators often suggest using strategies, such as the model-drawing, unitary, and guess-and-check strategies, to solve problems; the current study revealed that they might not be helpful for solving algebraic problems. The strategies used for solving arithmetic problems could not help the algebra novice students in Class 2 to achieve a high success rate when compared to the algebraic strategies used by the algebra proficient students in Class 1. Although model-drawing, unitary, and guess-and-check strategies are valid and helpful methods for students to gain an understanding of mathematical concepts, they are not as efficient as algebraic strategies in solving algebraic word problems. This finding supports the arguments of Fong (1994) and Jiang (2005, 2016): it is important for students in the transition period to gradually move from model drawing, unitary, and guess-and-check to algebraic strategies. Fong (1994) used several examples to show how to transition from the model drawing to algebraic strategies; the solutions in Figure 1 indicated that similar ways could be used to build the links between unitary and algebraic strategies. An example below shows how to help students to move from the guess-and-check to algebraic strategies.

In the solution shown in Figure 2, the student checks the constraint whether the total distance of the journey is 150 km, which is actually the basis on which most students using the algebraic method set up equation(s) ($15x+75(6-x)=150$, where x is set as the time she cycled). Researchers generally acknowledge that students have difficulties using unknowns to express other unknowns when moving from arithmetic to algebraic thinking (Bednarz & Janvier, 1996; Stacey & MacGregor, 1999). However, we argue that students might have greater difficulty choosing which relationship to base on for setting up an equation. For example, Problem 9 involves five relationships ($TD = D_1 + D_2$, $TT = T_1 + T_2$, $D_1 = S_1 \times T_1$, $D_2 = S_2 \times T_2$, and $TD = AS \times TT$). In the process of setting up the above equation, four of them except $TD = AS \times TT$ have been used to represent the other four unknowns (D_1, T_2, D_2, and TD) (i.e., $D_1 = S_1 \times T_1 = 15x$; $T_2 = TT - T_1 = 6-x$; $D_2 = S_2 \times T_2 = 75(6-x)$; and $TD = D_1 + D_2 = 15x+75(6-x)$). The students might have forgotten what relationships they have used for deducing other variables in this long process and be stuck on what relationship to base on for setting up the equation. If a student can produce a complete guess-and-check cycle as the one shown in Figure 2, the student can be guided to deduce other variables (Kieran, 1996) and eventually form an equation based on the constraint(s).

Figure 2
A Solution to Item 9 Using the Guess-and-Check Strategy

Guess					Check
T_1 (hours)	D_1 (km)	T_2 (hours)	D_2 (km)	TD (km)	
1	15	5	375	390	×
2	30	4	300	330	×
3	45	3	225	270	×
4	60	2	150	210	×
5	75	1	75	150	√

In the unit "Equations" of secondary school mathematics, the first lesson is "From Arithmetic to Equations." What relationship to use as the basis for setting up an equation is emphasized through a comparison between arithmetic and algebraic methods for solving word problems (Cai, 1998). The guess-and-check strategies could also be compared with algebraic strategies by emphasizing that the relationship used to form the equation in the algebraic method is usually the last constraint checked by the students in the process of using the guess-and-check method. This might be another way to help students to develop a better understanding of the connections between arithmetic strategies and algebraic strategies. Such an idea needs to be further investigated.

Discrepancies existed in the item difficulty measures of three items. The strategy analysis revealed that the use of formula as a special kind of arithmetic strategies could make items 10 and 11 easy. On the other hand, item 10 was relatively more difficult than item 11. This is probably because the complexity of computations involved in item 10 made the use of the guess-and-check strategy less effective, although this strategy is often taken as an effective problem-solving strategy for such problems.

Although the current study presents important new information explaining item misfit and DIF between the Singaporean and Chinese students found in the previous study (Jiang et al., 2018), limitations exist. First, only 14 word problems on speed were included in the current study. Further studies could include more problems, for example, the problems with the distance-time and/or speed-time graphs, in the test. Second, the current study collected data from Singaporean and Chinese students; further studies could be conducted with samples from other countries, in particular students from western countries. Third, more evidence from the teaching of mathematical problem solving in classrooms is needed for explaining the results obtained in the current study. Fourth, in addition to the traditional IRT model used in our previous study (Jiang et al., 2018) and the mixed IRT model used in the present study, future research can also adopt cognitive diagnostic models for analysis (Wang et al., 2016) to better understand students' cognitive processes so as to develop instruction tailored to students' needs and to improve student achievement. Despite these limitations, this study is the first that specifically focuses on word problems on speed and examines their item difficulty levels with students from Singapore and China, two high-performing countries in international mathematics study. We hope this study will bring more research to similar mathematics topics.

In summary, the analysis using the mixed IRT model provided more insights into cross-culture differences in mathematics achievement, which stem from curricular differences. The findings of this study have important implications for the validity and understanding of a potential source of measurement error in mathematical problem solving. The findings also have clear instructional implications for teachers with respect to instructional design. Lastly, the study provides insights and implications for designing and conducting cross-culture research.

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Appendix: The Mathematical Problem-Solving Test on Speed

A. Short-answer questions

1. A man drove at 72 km/h for 2 hours, then the distance he traveled was ______ km.
2. It takes a motorist ______ hours to travel 136 km at a speed of 68 km/h.
3. 3. Judy cycled 45 km in 3 hours, then her speed was ______ km/h.

B. Open-response questions

4. A man went to visit his friend who lived 27 km away. After walking at a speed of 6 km/h for $1\frac{1}{2}$ hours, he finished the remaining journey at a jogging speed of 12 km/h and got to his friend's house at the appointed time. Find his average speed for the whole journey.
5. Mike made a journey from City P to City Q. In the first hour, he covered $\frac{1}{3}$ of it. In the second hour, he covered $\frac{1}{5}$ of the whole journey. Finally, he took 2 hours to finish the remaining journey at a speed of 42 km/h. Calculate his average speed for the whole journey.
6. Town A and Town B are 20 km apart. Mike left Town A at 8.00 a.m. and cycled at 16 km/h towards Town B. 10 minutes after Mike started, Bill cycled from Town B towards Town A at 15 km/h. (a) At 8.30 a.m., how far was Mike away from Town A and how far was Bill away from Town B? (b) Did they meet up at 8.30 a.m.? If not, how far were they apart from each other at 8.30 a.m.?
7. Mike made a journey from City P to City Q. In the first half an hour, he covered $\frac{1}{7}$ of it. In the second half an hour he covered $\frac{1}{3}$ of the remaining journey. Finally, he took another half an hour to finish the journey at a speed of 72 km/h. Calculate his average speed for the whole journey.
8. Yesterday afternoon, Mike drove to the kindergarten to take back his son. It was raining on the way to the kindergarten, so he had to drive at a slow speed of 39 km/h, and it took him $1\frac{1}{3}$ hours to get there. He returned soon at a faster speed along the same way. When he came back home, his wife told him that his average speed for the round trip was 52 km/h (ignoring the time spent in the kindergarten). Find his speed on the way back.
9. On Sunday, Judy went to see her grandma who lived 150 km away. After cycling at an average speed of 15 km/h for a few hours, she got tired and took a lift from the passing truck. The truck's average traveling speed was 75 km/h. When she got to her grandma's house, she checked the time and knew that the trip took her 6 hours. Find the time she spent cycling.
10. Two places R and S were 300 km apart. Mike left R and drove at 84 km/h towards S. At the same time, Bill left S at 60 km/h and drove towards R. How long did they take to meet?
11. Two motorists, Jack and John, are having a race in the athletic track. John starts first. Half an hour later, Jack begins to chase after him. John's driving speed is 84 km/h, and Jack's driving speed is 90 km/h. How long does Jack take to catch up with John?
12. On Sunday, the students of one class went out for a picnic. At first, they traveled in the crowded city for 40 minutes at a speed of 27 km/h. Then they covered $\frac{1}{5}$ of the remaining journey at a speed of 54 km/h. Finally, they traveled $\frac{2}{3}$ of the whole journey in one hour on the highway. Calculate the average speed for the whole journey.
13. Sunday morning, Rebecca and her parents went out to enjoy the natural scenery. On the way to the destination, they traveled at a slow speed of 40 km/h. On the way back, they drove at a faster speed of 120 km/h. When they came back home, they found that they had been out for 2 hours. Find the average speed for this round trip (ignoring time at the destination).
14. On the first day of this new term, Teacher Lee went to the bookshop to pick up the ordered textbooks. On the way to the bookshop, his speed was as slow as 24 km/h because of the heavy traffic. On the way back, the traffic was light, so he took only one hour. If the average speed for the round trip was 36 km/h, find the speed on the way back (ignoring time spent in the bookshop).